Problem 16

Can $y = \sin(t^2)$ be a solution on an interval containing $t = 0$ of an equation
$y'' + p(t)y' + q(t)y = 0$ with continuous coefficients? Explain your answer.

Solution

Because the coefficients, $p(t)$ and $q(t)$, are continuous, there exists a unique solution to the ODE
over some interval containing $t = 0$. The function $y = 0$ satisfies the ODE along with the initial
conditions, $y(0) = 0$ and $y'(0) = 0$, so $y = \sin(t^2)$ cannot also satisfy it and these conditions.